1. Fibonacci numbers = 0, 1, 1, 2, 3, …

Base: F12 =

Inductive step:

Assume f12+f22+...+fn2 =fnfn+1

Prove f12+f22+...+fn2 + fn+12=fn+1fn+2

LHS = f12+f22+...+fn2 + fn+12 By IH

= fnfn+1 + fn+12

= fn+1 (fn + fn+1)

= fn+1fn+2  by Fibonacci definition

= RHS

1. Base n = 0

State Machine is in state 0 after 0 steps.

Induction step:

Assume that k is divisible by 4 if and only if state machine is in state 0 after k steps.

Prove that (k+1) is divisible by 4 if and only if state machine is in state 0 k steps.

K mod

State machine is in state 0 after k steps, by IH. So, it will be in state 1 or 2 after k+1 step. Since k is divisible by 4, k+1 would have a remainder when divided by 4, so (k+1) assertion holds.

1. P(1) and P(2) are true

Therefore, p(k) is true when k is 1, 2, 4, 5, 7 and any other number that is not multiple of 3.

Therefore, p(k) is true when k is 1, 2, 3, 4, and any other positive integers.

1. Start (0,0)

Steps (-1,+3), (+2,-2) and (+4,0)

So, robot can only be at position (x, y) which will always divisible by 4

Base case:

P(0) = 0 which is divisible by 4

Induction step:

Case 1: (-1, +3)

and it is divisible by 4

Case 2: (+2, +2)

and it is divisible by 4

Case 3: (+4, 0)

and it is divisible by 4

(2,0)

and it is not divisible by 4

Therefore, robot will never get to (2, 0) position.

1. Assume

Let k be an element of S.

P(k) is true if k is a power of 2 and false otherwise.

Let k + 1 be an element of S too.

We know that k+1 will always be greater than k and p(k+1) is true because all integers of S are true.